Asymmetric Inventory Dynamics and Product Market Search

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Abstract

Why does inventory investment on average account for 72% of GDP decline during recessions but only 8% during expansions? Why does the aggregate inventory-sales ratio cease to be countercyclical since the 1990s but still lag GDP for four quarters? These newly documented stylized facts pose challenges to existing macroeconomic inventory models and cast doubts on important conclusions drawn from these models. In this paper, I show that incorporating product market search friction into a standard inventory model can address these stylized facts. Product market search enhances firms' asymmetric trade-off between accumulating inventory and adjusting markup, and thereby generates strongly asymmetric shares of inventory investment in GDP movements. Product market search also generates the lagging inventory-sales ratio because households' procyclical effort to search for varieties increases(decreases) sales as well as inventory stock at the early stage of expansions(recessions). Its effects, however, are later eclipsed by heightened(lowered) return on holding inventory which only increases(decreases) inventory stock but not sales. The model is disciplined by micro evidence provided by recent empirical studies, and its behavior is consistent with inventory and business cycle stylized facts in the US. Additionally, the model is broadly consistent with observed business cycle asymmetries in output, employment, and markup.

JEL classification: E32; E22

Keywords: Inventory; Product Market; Business Cycle Asymmetry; Search and Match Friction

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"...indeed, to a great extent, business cycles are inventory fluctuations."

Blinder (1981)

"Everything that needs to be said has already been said. But since no one was listening, everything must be said again."

Andre Gide

1 Introduction

Inventory investment, defined as the change of aggregate inventory stock over a certain period, is a volatile component of GDP. As shown in Figure 1, even though it amounts to less than 1% of GDP, inventory investment explains close to 25% of GDP volatility. Its ability to explain GDP volatility is greater than other important variables such as fixed investment, consumption, and net export. Strikingly, inventory investment accounts for 72% of GDP declines in post-war US recessions. Macroeconomists should not ignore inventory.

However, inventory is not a core feature in most modern macroeconomic models, perhaps due to a lack of consensus on how inventory should be modeled. Researchers disagree on which friction[1] is the most appropriate for incorporating inventory into models[2]. The primary reason for this disagreement is that each model fails to account for some stylized facts of inventory behavior.

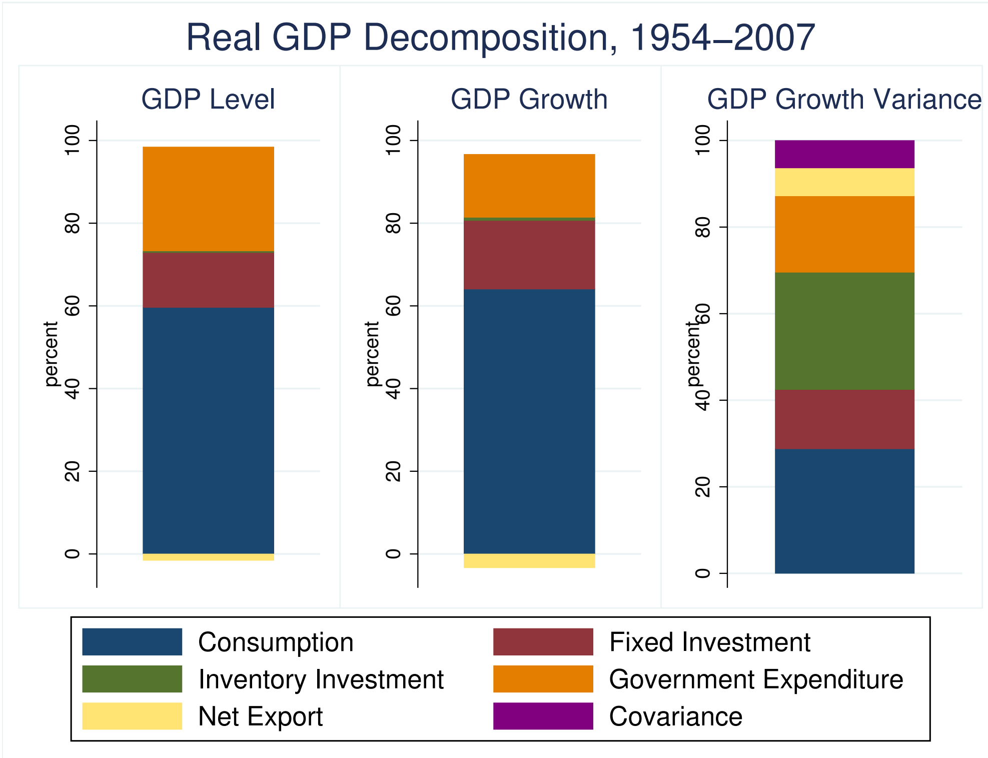
With the goal of building better inventory models, this paper documents and explains two novel stylized facts: the asymmetric importance of inventory investment and the lagging nature of inventory-sales ratio. These newly documented facts are crucial for two reasons. First, inventory investment explains a much larger share of GDP movement in recessions than expansions. However, unconditional statistics largely driven by dynamics during expansions[3] are commonly used in existing studies as stylized facts. Ignoring the asymmetry can mask the actual role of inventory. Second, many models treat the countercyclicality of inventory-sales ratio as a stylized fact to match quantitatively and draw conclusions on important

[1] In a frictionless model such as the off-the-shelf Real Business Cycle model in King and Rebelo (1999), agents would hold zero inventory at steady-state due to the problem of return rate dominance.

[2] Two primary candidates are stockout avoidance (Kahn (1987, 1992); Wen (2005, 2011)) and nonconvex adjustment costs (Blinder and Maccini (1991); Caplin (1985); Khan and Thomas (2007))

[3] For a more complete documentation of stylized facts differences across regimes, see Chen (2017)

Figure 1: Disproportionate Importance



NOTE: US Real GDP Decompositions. Data taken from St. Louis FED FRED database. All shares are averages across the sample period. The first, second, and third columns show the decomposition of GDP levels, GDP growth rates, and variances of GDP growth rates, respectively. The key take-away is how inventory investment (green bar) is negligible in the level but dominant in the variance decomposition. This figure extends the one produced in McMahon (2012).

issues. Examples include the transmission mechanism of monetary policy (Kryvtsov and Midrigan (2012)), cyclicality of markup (Bils and Kahn (2000); Bils (2004)), and the recent structural change of US business cycles (McConnell and Perez-Quiros (2000); Sarte et al. (2015)). However, the inventory-sales ratio has become procyclical since the 1990s while it consistently lags GDP for four quarters throughout the post-war period (1947-2017). Ramey and West (1999); Jung and Yun (2005); Maccini et al. (2015) report similar findings from studying impulse responses to different shocks. This lagging relationship, which could shed light on propagation mechanisms in inventory models, remained unexplained until now. In sum, I argue that these two stylized facts are crucial for building better models and understanding inventory in macroeconomics.

I start with an off-the-shelf inventory model (most closely related to Kahn (1992); Wen (2011)), then augment it with product market search friction. Once the model is disciplined with micro evidence from recent empirical studies on household shopping behavior and inventory data, it generates the two newly documented stylized facts along with existing ones. In addition, the model is broadly consistent with observed business cycle asymmetries in output, employment, and markup.

Product market search emphasizes the searching and matching process between households and products. This friction introduces an extensive margin of demand uncertainty from the firms' perspective, while standard models only feature an intensive margin resulting from an exogenous i.i.d. demand shock. While standard models with stockout avoidance motive alone exhibit built-in asymmetric trade-off between inventory holding and markup adjustment, this new extensive margin amplifies the asymmetry by providing an extra force that affects the stockout probability. Product market search also generates the lagging inventory-sales ratio because households' procyclical effort to search for varieties increases(decreases) sales and inventory stock at the early stage of expansions(recessions). Its effects, however, are later eclipsed by heightened(lowered) return to holding inventory, which only increases(decreases) inventory stock but not sales.

This paper connects two active bodies of research: inventory[4] and product market search friction. Much like how search friction in the labor market explains the existence of unemployment, search friction in the product market helps explain large aggregate inventory stock, which is the result of excess supply relative to demand. Surprisingly, few research papers make this connection except den Haan (2013)[5].

This paper adds to the effort of building macroeconomic inventory models based on the stockout avoidance motive. First proposed in Kahn (1987), this modeling approach has been introduced to general equilibrium models in Jung and Yun (2005), Wen (2011) and Kryvtsov and Midrigan (2012), due to its ability to match empirical regularities of inventory dynamics. Results in this paper further attest the merits of stockout avoidance motive, as it can generate two more documented stylized facts when combined with product market search friction.

This paper is also closely related to a growing body investigating the implications of product market search friction. My approach to model product market search follows Huo and Ríos-Rull (2013). Bai et al. (2012) shows that their model economy's responses to de-

[4]For comprehensive reviews of the macroeconomic inventory literature, see Blinder and Maccini (1991) and Ramey and West (1999).

[5]The search friction in den Haan (2013) is modeled without explicit micro-foundations.

mand shocks and productivity shocks are very similar when search friction is introduced. Similar to my results, product market friction significantly amplifies and propagates productivity shocks in Petrosky-Nadeau and Wasmer (2015). In contrast to this line of literature, my paper incorporates inventory and investigates the link between inventory and product market search friction.

The plan of this paper is as follows: Section 2 documents two new stylized facts, Section 3 lays out the model, Section 4 conducts quantitative analysis to examine the model's ability to explain the stylized facts, Section 5 dissects the mechanism and provide intuitions on how the model generates the stylized facts, and Section 6 concludes.

2 Two New Facts About Inventory Dynamics

2.1 Fact 1: Empirical Asymmetry of Inventory Investment

Large decumulations of inventory stock coincide with recessions. In contrast, accumulations do not coincide with expansions. Being a minute fraction of GDP levels, inventory investment accounts for on average 72% of GDP declines in recessions[6] but only 7% of GDP increases in expansions. For all post-war business cycles, the fractions of GDP movements attributable to inventory investment are displayed in Table 1. Inventory investment clearly plays a larger role in peak-to-trough movements than in trough-to-peak movements.

An alternative way to characterize the asymmetry in inventory investment is to look at the skewness of inventory-investment-to-GDP ratio. Negative skewness indicates more frequent decumulation of inventory relative to GDP than accumulation. Using quarterly data from 1954 to 2017[7], we can see from Figure 2 that this ratio has a longer left tail (skewness -0.33). The left tail features frequent negative values more than twice the mean.

Since the seminal work of Feldstein and Auerbach (1976), economists have argued for the importance of inventory using its peak-to-trough behavior, yet the overall asymmetric behavior is unexplored. Existing studies fail to explain why trough-to-peak inventory investment only accounts for a small share of GDP expansions. What's more troubling is that most macroeconomic inventory models are empirically disciplined with unconditional statistics of inventory variables, which are primarily driven by dynamics during expansions.

[6] As defined by the National Beaureau of Economic Research (NBER).
[7] See the appendix for data sources.

In contrast, inventory seems to be playing a more significant role in recessions.

2.2 Fact 2: The Lagging Inventory-Sales Ratio

One important dimension of inventory dynamics has remained unexplored: inventory-sales ratio consistently lags the GDP by four quarters[8]. Most inventory models are not consistent with this fact (e.g. Wen (2011)) as they fail to generate hump-shaped responses of inventory stock that drive this lagging relationship. From Figure 4, we can see that the inventory-sales ratio comoves reasonably well with the GDP at the fourth lag. This is further confirmed by an inspection of the cross-correlation of the two series: in Figure 5 we can see clearly that the strongest positive correlation (0.4) happens at the fourth lag.

Inventory-sales ratio, defined as the ratio between end-of-period inventory stock and sales during a period, is an important variable extensively studied by practitioners and academic researchers alike. Its countercyclicality has led authors (e.g. Bils and Kahn (2000); McConnell and Perez-Quiros (2000); Bils (2004); Kryvtsov and Midrigan (2012); Sarte et al. (2015)) to draw conclusions about important issues including the transmission mechanism of monetary policy and the source of structural break in US business cycle dynamics.

The inventory-sales ratio ceases to be countercyclical around the early 1990s, but the lagging relationship remains true. The sample correlation between the inventory-sales ratio and the GDP is -0.24 before 1992. After 1992, this correlation becomes 0.24. Using a 40-quarter moving window estimation of sample correlation, we can see that the correlation is positive in the early sample (approximately the period before the Korean War) and the recent decades while it is negative in between.

The key take-away here is that we should focus on the lagging nature of the inventory-sales ratio instead of its countercyclicality. Even though important conclusions predicated on the countercyclicality of inventory-sales ratio have been drawn in the literature, this cyclicality has reversed in recent decades. In contrast, the lagging relationship between the inventory-sales ratio and the GDP has been stable across the entirety of the post-war sample. This relationship is consistent with empirical findings where inventory stock displays hump-shaped responses to common shocks (Ramey and West (1999); Jung and Yun (2005); Maccini et al. (2015)). It is not until now that the lagging nature of the inventory-sales ratio is investigated in theoretical models.

[8]Both variables Hodrick and Prescott (1997) filtered with smoothing parameter 1600

Table 1: Contributions of Inventory Investment to GDP Changes in Post-war US Business Cycles

(a) Peak to Trough Declines, Annualized Billions of Dollars

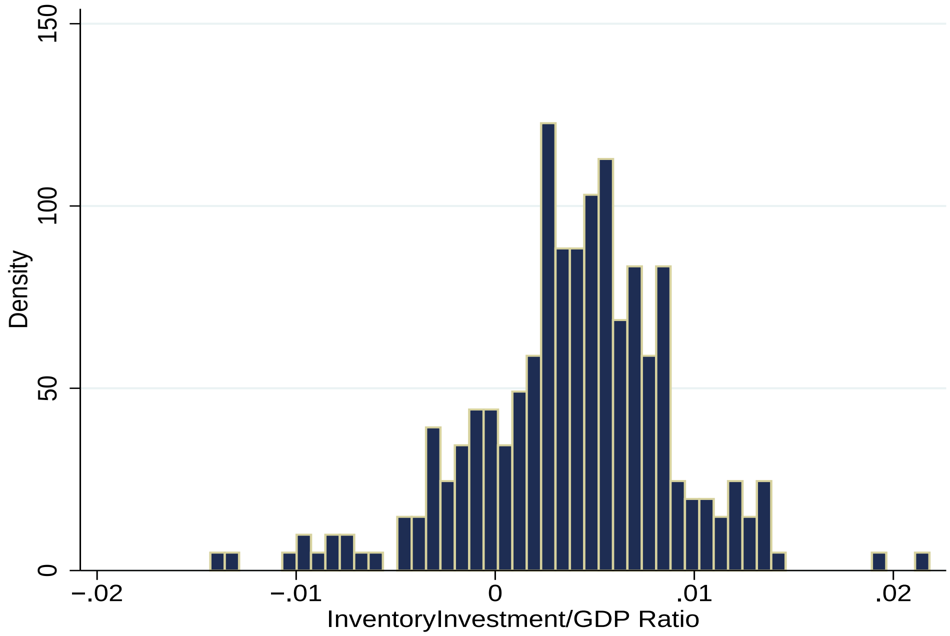
Date	Inventory Investment	GDP	Share
1948:4-1949:4	-40.66	-30.68	132%
1953:2-1954:2	-24.47	-62.77	39%
1957:3-1958:2	-21.19	-84.98	25%
1960:2-1961:1	-21.38	-9.06	236%
1969:4-1970:4	-35.84	-7.19	498%
1973:4-1975:1	-80.06	-169.95	47%
1980:1-1980:3	-67.26	-142.02	47%
1981:3-1982:4	-120.51	-169.73	71%
1990:3-1991:1	-46.87	-118.38	39%
2001:1-2001:4	-24.09	-40.20	60%
2007:4-2009:3	-213.07	-636.23	33%
			avg. 72%

(b) Trough to Peak Increases, Annualized Billions of Dollars

Date	Inventory Investment	GDP	Share
1949:4-1953:2	33.28	588.80	6%
1954:2-1957:3	20.94	345.25	6%
1958:2-1960:2	22.89	320.36	7%
1961:1-1969:4	30.15	1613.21	2%
1970:4-1973:4	76.25	754.12	10%
1975:1-1980:1	33.01	1232.47	3%
1980:3-1981:3	115.93	279.97	41%
1982:4-1990:3	84.35	2490.81	3%
1991:1-2001:2	7.16	3844.74	0.1%
2001:4-2007:4	120.34	2286.52	5%
			avg. 8%

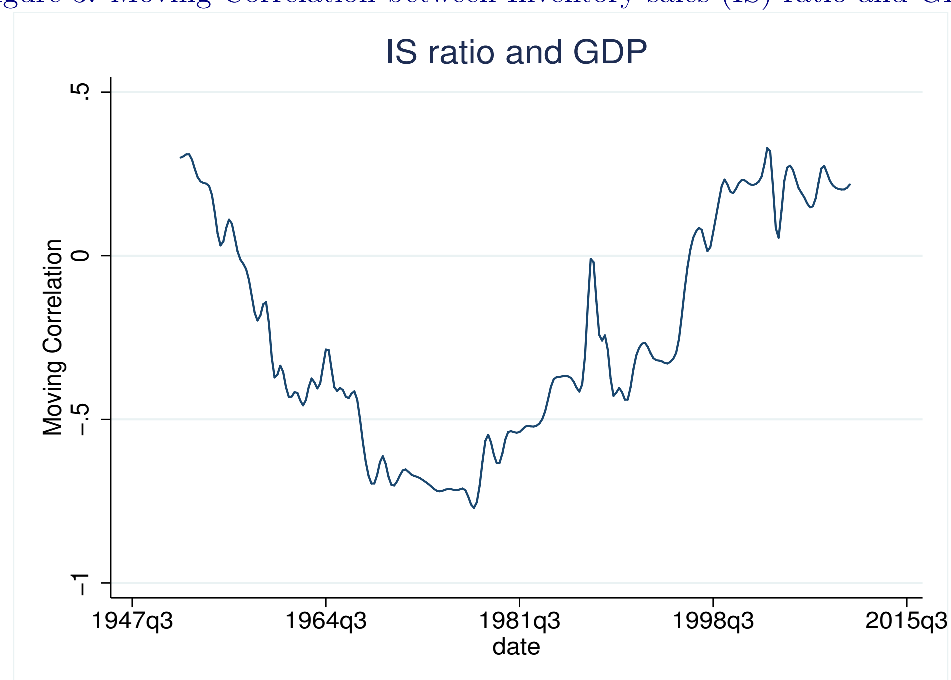
NOTE: The entries in this table denote cumulative decline(increase) from the peak(trough) to the trough(peak) of each cycle in units of annualized billions of 2009 dollars. The last column simply counts the ratio of the second to the third column. All data are acquired from the FRED database.

Figure 2: Histogram of Inventory Investment Relative to GDP



NOTE: Quarterly US data taken directly from Real GDP series published by the Bureau of Economic Analysis. The ratio is HP filtered with smoothing parameter 1600. The sample spans from 1954 to 2017 and the sample skewness is -0.33.

Figure 3: Moving Correlation between Inventory-sales (IS) ratio and GDP



NOTE: 40 quarters moving windows contemporaneous correlation. Correlation for the sample before 1992 is -0.24. Correlation for the sample after 1992 is 0.23

Figure 4: Inventory-Sales Ratio Lags GDP by Four Quarters

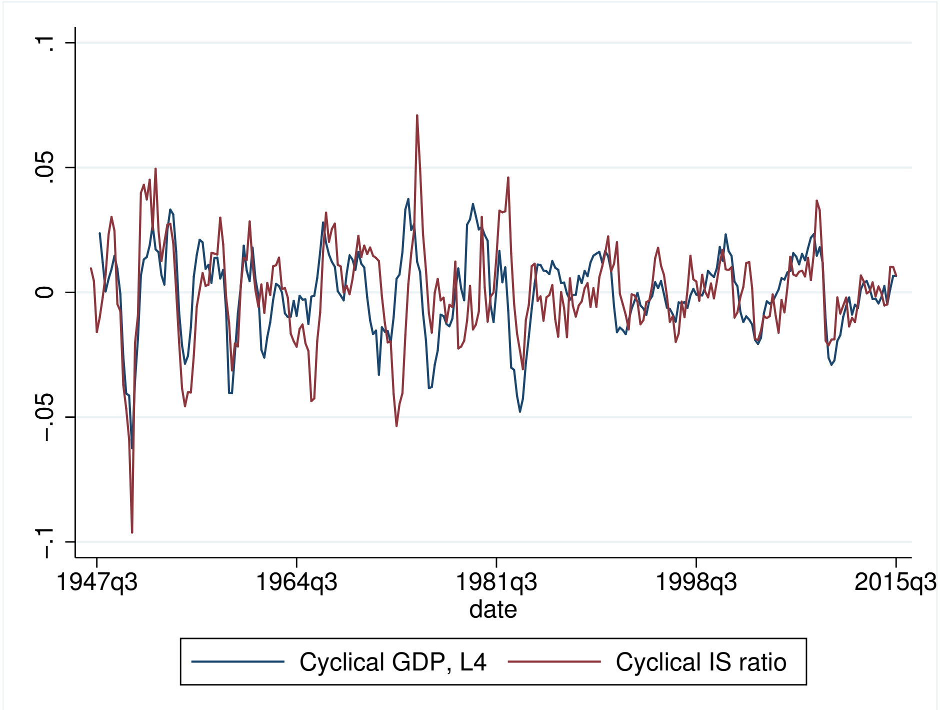
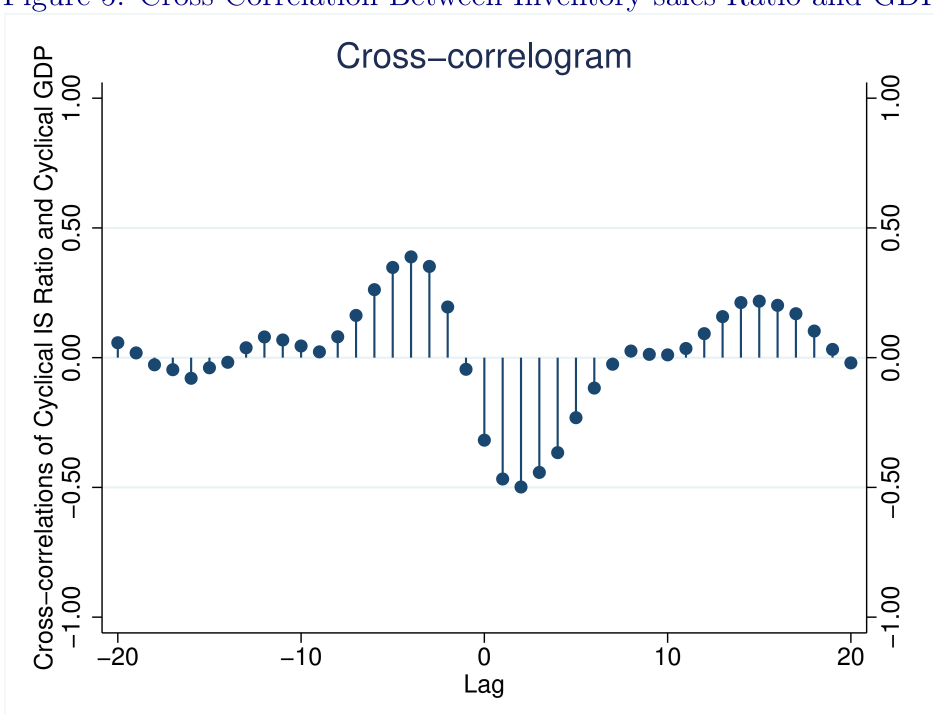


Figure 5: Cross-Correlation Between Inventory-sales Ratio and GDP



3 Model

In this section, I describe a dynamic stochastic general equilibrium(DSGE) model to investigate the asymmetry in inventory dynamics and the lagging property documented in previous sections. The first order conditions and their derivations can be found in the appendix.

3.1 Environment

Time is discrete and denoted by $t \in \mathbb{N}$.

Agents The model economy is populated with four types of agents: households, intermediate good producers, variety good producers, and final good producers. All agents are normalized to be measure one.

Markets Households are endowed with one unit of time that they can supply to the intermediate producers in exchange for wage income. Only intermediate producers can utilize labor for production and the labor market is perfectly competitive. Intermediate good is sold in a perfectly competitive market where variety good producers purchase intermediate good in order to produce differentiated goods indexed by variety $i \in [0, 1]$. To fix ideas, I assume the variety good producers paint the otherwise identical intermediate good with different colors, indexed by i, to create product differentiation. The market for each variety good is monopolistic competitive as in the classic formulation of Dixit and Stiglitz (1977). Each variety can only be produced by one firm thus I will index both of them by variety i. Final good producers purchase varieties from these monopolistic competitive markets and sell their homogeneous products to households in a perfectly competitive market. Finally, I assume a single corporation owns all producers and one unit of equity is issued and traded on a stock market. The price of this stock is used as the numeraire.

Product Market Search Households exert efforts to search for variety in the final consumption bundle. However, they can't reach all varieties (measure one) due to search and match friction (for a textbook treatment of the friction see Pissarides (2000)). Denote the time t aggregate measure of search effort by households to be D_t, then the quantity of effort-variety matches is given by a matching function:

$$x_t = G(D_t, 1) \tag{1}$$

where the second argument is one because all variety producers participate in the search and match process. I assume the matches are uniformly distributed on each household, therefore each unit of search effort acquires $\Psi_{D,t}$ units of varieties:

$$\Psi_{D,t} \equiv \frac{x_t}{D_t}. \tag{2}$$

3.2 Household

There is a unit measure of identical households in the economy. The representative household maximizes the expected lifetime utility and solves the following problem, taking as given initial stock holding a_{-1} and sequences of prices $\{w_t, \bar{P}_t, \Psi_{D,t}\}_{t=0}^{\infty}$:

$$\max_{\{c_t, a_{t+1}, d_t, n_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t x_t^{\rho}, d_t, n_t)$$

$$s.t. \quad \bar{p}_t x_t c_t + a_{t+1} \leq a_t(1 + \Pi_t) + w_t n_t \tag{3}$$

$$x_t = \Psi_{D,t} d_t \tag{4}$$

where $U(\cdot)$ is the period felicity function, c_t the period t average consumption index, x_t the number of variety in the consumption bundle, and $\bar{p}_t$ the average variety good price index. Lastly Π_t denotes the profit flow returned from ownership of the corporation.

Households decide how much variety x_t they would consume according to (4). Search effort d_t incurs disutility but brings about matches with variety producers at the rate of $Psi_{D,t}$ which the household take as given. The degree of "love for variety", or the inverse of elasticity of substitution across varieties, is controlled by the parameter $\rho > 1$. Final consumption bundle $c_t x_t^{\rho}$ consists of x_t varieties and each variety averages c_t amount of consumption.

This formulation of household's problem highlights the choice for varieties and search effort similar to that in Huo and Ríos-Rull (2013). An alternative formulation, where the household and final good producers are lumped into one agent and choose consumption of each variety along with the number of varieties, is located in the appendix. These two formulations yield identical optimality conditions and interpretations, but I separate the problems of households and final good producers for the simplicity of exposition.

3.3 Final Good Producer

Final good producers pack x_t measures of varieties, dictated by the household, into final goods. For tractability, I assume all final good producers always purchase from varieties with index from 0 to x_t, but this fact is unknown to the variety good producers. The representative final good producer solves the following problem:

$$\max_{c_t, \{c_{i,t}\}_{i=0}^{x_t}} \bar{p}_t x_t c_t - \int_0^{x_t} p_{i,t} c_{i,t} di$$

$$s.t. \quad c_t = \left(\frac{1}{x_t} \int_0^{x_t} v_i^{1-\frac{1}{\rho}} c_{i,t}^{\frac{1}{\rho}} di \right)^{\rho} \tag{5}$$

$$c_{i,t} \leq z_{i,t}. \tag{6}$$

Taking as given the competitive price $\bar{p}_t$ of final good, prices $\{p_{i,t}\}_{i=0}^{x_t}$ of variety goods, available quantities $\{z_{i,t}\}_{i=0}^{x_t}$, and the required measure of varieties x_t, the final good producer decides the quantities it will purchase for each variety $\{c_{i,t}\}_{i=0}^{x_t}$. It produces the final good with technology described by (5) using different varieties $\{c_{i,t}\}_{i=0}^{x_t}$. The productivity of variety i is buffeted by an idiosyncratic shifter v_i which is known to the final good producer at the time of decision. The shifter v_i is an random variable distributed identically and independently across time t and variety i. I assume the sales of each variety cannot exceed z_i, the amount made available by the variety i producer. Stock-out happens when the price of variety i is low enough such that the availability constraint (6) becomes binding.

The decisions for $c_{i,t}$ generate demand curves for variety i:

$$c_{i,t} = \min \left\{ z_{i,t}, v_{i,t} c_t \left(\frac{p_{i,t}}{\bar{p}_t} \right)^{\frac{\rho}{1-\rho}} \right\} \tag{7}$$

which the variety i producers take as given (more details in the following subsections). See Figure 8 for a visual representation of this curve. Final good producers purchase variety i goods according to the price relative to an average price index and the average consumption index c_t, which I can interpret as a measure of market size, until the demand reaches maximum availability $z_{i,t}$.

The average price index satisfies:

$$\bar{p}_t = \left[\frac{1}{x_t} \int_0^{x_t} v_{i,t} (p_{i,t} + \mu_{i,t})^{\frac{1}{1-\rho}} \right]^{1-\rho} \tag{8}$$

where $\mu_{i,t}$ is the Lagrange multiplier associated with constraint (6). The term $(p_{i,t} + \mu_{i,t})$ is the reservation price that final good producers are willing to pay. In the case where stockout happens, $\mu_{i,t}$ is nonnegative, indicating that the market price is lower than the reservation price. In other words, realized demand is larger than what's made available. Alternatively, when stockout doesn't happen, constraint (6) does not bind and thus $\mu_{i,t} = 0$. Price index $\bar{p}_t$ captures the average reservation price after adjusting for productivity shifter v_i. Therefore $\bar{p}_t$ is the relevant quantity in the determination of variety i demand, Equation 7.

3.4 Variety Producer

For the ease of exposition I formulate the variety producer's problem as a Bellman's equation:

$$\begin{aligned}
\mathscr{V}(e_i) = & \max_{y_i, p_i, e_i^{'}} -p_M y_i + x \int \left\{ c_i p_i + \mathbb{E} m^{'} \mathscr{V}(e_i^{'}) \right\} F_v(dv_i) + (1-x)\mathbb{E} m^{'} \mathscr{V}(e_i^{'}) \\
s.t. \quad & c_i = \min \left\{ z_i, v_i \left(\frac{p_i}{\bar{p}} \right)^{\frac{\rho}{1-\rho}} \bar{c} \right\} \\
& z_i = e_i + y_i \qquad (9) \\
& e_i^{'} = \begin{cases} (1-\delta_e)\left[e_i + y_i - c_i \right] & \text{“matched”} \\ (1-\delta_e)\left[e_i + y_i \right] & \text{“unmatched”} \end{cases} \qquad (10)
\end{aligned}$$

where I suppress the subscript t for the sake of simplicity and use $'$ to denote variable at the next period $(t + 1)$.

At the beginning of a time period, the variety i producer starting with e_i amount of inventory stock decides on the price of its own good (p_i), new order y_i, and inventory at the end of period $e_i^{'}$ to maximize its value $\mathscr{V}$. The goods available for sale z_i is the sum of existing inventory e_i and new orders y_i. Inventory stock carried over to the next period is simply the amount of goods available minus sales and depreciation (Equation 10). The price of new orders (of intermediate good) is simply P_M and all future payoffs are discounted with household's stochastic discount factor m'.

With probability x, it matches with final good producers and can make sales according to final good producers' demand schedule Equation 7. With probability $1-x$, the variety producer is unmatched and cannot make any sales at all. If the variety producer is matched, then the idiosyncratic demand shock v_i[9], with i.i.d. distribution F_v , is revealed and sales c_i is determined. It is clear from the Bellman's equation that the ex-post realizations of sales and future inventory stock are uncertain at the time of decisions, therefore the producer maximizes expected values (taking expectation over the discrete realizations of "matched" versus "unmatched" and over the continuous random variable v_i) of these quantities.

Similar to Wen (2005, 2011); Kryvtsov and Midrigan (2012), I assume the producer has to decide on price and new orders before knowing whether it is matched with consumers and the realization of v_i. Proposed first in Wen (2005), having to make decisions before observing v_i generates incentives to hold inventories to guard against situations when demand shock v_i is so high that stockout happens. What's new here is that the variety producer faces another layer of uncertainty in decision-making: it cannot sell with probability $(1-x)$. When the variety good producer is not matched with buyers, the realization of demand shock v_i ceases to matter.

The optimal price decision must satisfy:

$$p_i = \frac{\epsilon_i}{\epsilon_i - 1}(1-\delta_e)\mathbb{E}m' P'_M \tag{11}$$

where the price elasticity of expected sales ϵ_i is given by:

$$\epsilon_i = \frac{\rho}{\rho-1} \frac{\int_0^{v_i^*} c_i(p_i, n_i, v_i) F_v(dv_i)}{\int_0^{v_i^*} c_i(p_i, n_i, v_i) F_v(dv_i) + [1 - F_v(v_i^*)]\,[e_i + y_i]}. \tag{12}$$

and

$$v_i^* = \frac{z_i^*}{\overline{c}} \left(\frac{p_i^*}{\overline{p}}\right)^{\frac{\rho}{\rho-1}} \tag{13}$$

In other words, price is the product of shadow markup $\frac{\epsilon_i}{\epsilon_i-1}$ times the discounted value of intermediate price tomorrow. The latter is an appropriate concept for "marginal cost" because it represents the replacement cost of the marginal unit of sales. Since unsold goods

[9] It is same the variable called "productivity shifter" in Section 3.3. However from the perspective of variety good producer it is a demand shifter because v_i shifts the demand curve faced by variety producer.

get carried over to the future, the opportunity cost of selling the marginal unit of variety good is the cost to replace it tomorrow. The shadow markup depends on the price elasticity of the expected demand curve, e_i, which in turn depends on the fraction of expected sales associated with stockout happening. More frequent stockout (due to lower v_i) leads to less price elasticity ϵ_i and thus higher shadow markup.

The optimal amount availability z_i is determined by the following trade-off:

$$x\left[1-F^{v}(v_i^*)\right]\left[p_i-(1-\delta_e)\mathbb{E}m'p_M^{'}\right]=p_M-(1-\delta_e)\mathbb{E}m'p_M^{'}. \tag{14}$$

The left hand side (LHS) of the equation is the expected marginal benefit of having an extra unit of variety good available for sale. If stockout happens, then the marginal unit of variety good available becomes sales, increasing the producer's value by the difference between price and the discounted value of inventory tomorrow (this marginal unit of variety good becomes sales instead of inventory). The right hand side (RHS) of this equation is the inventory holding cost. When stockout doesn't happen, the marginal unit of order becomes $(1-\delta_e)$ units of end-of-period inventory stock due to depreciation. Instead of paying P_m for this marginal unit which yields no benefit, the variety producer can pay a discounted value of $\mathbb{E}m'P_M'$ for $(1-\delta_e)$ units in the next period and save the difference $P_m-(1-\delta_e)\mathbb{E}m'P_M'$. All in all, the producer faces a trade-off between inventory holding cost and stockout avoidance benefit.

3.5 Intermediate Producer

There are measure one of intermediate producers who convert labor inputs n_t into homogeneous intermediate goods with production function $F(A_t, n_t)$, where A_t is the exogenous productivity level. They operate in a perfectly competitive market in which buyers are the variety producers as described in previous subsections. Wage rate is w_t and the goods are sold at the prevailing market price of $p_{M,t}$. The representative intermediate producer's problem is therefore static and simple:

$$\max_{n_t} p_{M,t}F(A_t, n_t)-w_tn_t \tag{15}$$

Assuming the intermediate goods are produced by agents other than variety good producers themselves makes the model significantly more tractable. If each variety good producer possesses the technology to produce, then those who have higher inventory stocks would face lower marginal cost of production because the production technology is convex. Each variety good producer would then make different pricing and ordering decisions based on their inventory stock levels. This introduces persistent heterogeneity ex-ante and makes analytical aggregation of individual firm's decisions impossible. A solution method similar to Krusell and Smith (1998) is required, and analysis of the model would become orders of magnitude harder.

3.6 Equilibrium

The definition of equilibrium is standard: all agents solve their optimization problems and all markets clear given exogenous processes. For a detailed definition and first order conditions the reader is referred to the appendix. I focus on a symmetric equilibrium where all variety producers choose the same price and the same amount of goods available: $p_{i,t} = p_t^*, z_{i,t} = z_t^*, \forall i$. Why do these heterogeneities across varieties vanish? Inspecting Equation 11 and Equation 14 reveals the answer. Since all variety producers face exactly the same distribution of $v_{i,t}$, the same process of $P_{M,t}$, the same process of SDF m_{t+1}, the same probability to be matched with buyers x_t, and the same indexes $\overline{c_t}, \overline{p_t}$, they are making the same decisions. Similar results also emerge in Wen (2005, 2011).

4 Quantitative Analysis

In order to understand the behavior and performance of the model I need to specify functional forms of key relationships and calibrate the model to reasonable targets. The model is solved using third-order perturbation method to properly gauge the asymmetric dynamics in the model[10]. Within this reasonable framework, I then investigate whether the model can generate the newly documented facts in this paper along with the traditional set of stylized facts. Additionally, I investigate whether the model generates common business cycle asymmetries in output, employment, and markup.

[10]For comparisons of nonlinear solution methods for DSGE models including perturbation methods, see Aruoba et al. (2006)

4.1 Functional Forms

I assume the utility function of the representative household to exhibit the following functional form, with average consumption c, varieties x, search effort d, and labor n as arguments:

$$u(cx^\rho, d, n) = \log\left(cx^\rho - \zeta\frac{n^{1+\upsilon_n}}{1+\upsilon_n} - \xi d\right) \tag{16}$$

which is a generalized form of utility function first proposed in Greenwood et al. (1988). Huo and Ríos-Rull (2013) also adopts a similar utility function. my choice of utility function generates positive comovements among varieties x_t, search effort d, and average consumption level c. These positive comovements are consistent with recent empirical studies in household shopping behavior and product market search (Broda and Weinstein (2010); Li (2012)).

I assume the Cobb-Douglas production function is used by intermediate good producers. Production inputs are land and labor. Total factor productivity is denoted by A. I assume the labor share to be $1-\alpha$ and land share to be α. I normalize the land input to one constantly, therefore the production function is given by:

$$F(A, n) = An^{1-\alpha}. \tag{17}$$

Following Wen (2011), I assume the idiosyncratic demand shock follows a Pareto distribution with location parameter $\underline{v}$ and shape parameter σ_v:

$$F_v(v) = 1 - \left(\frac{v}{\underline{v}}\right)^{-\sigma_v}. \tag{18}$$

The matching function that determines the measure of household-variety matches is taken from den Haan et al. (2000) to guarantee the varieties match is between zero and one:

$$G(d, 1) = \frac{d}{(d^\iota + 1)^{1/\iota}} \tag{19}$$

where ι determines the elasticity of varieties with respect to search effort.

4.2 Calibration Strategy

I assume one discrete time period in the model represents one quarter. Since I am interested in the dynamic behavior of the model, its parametrization is disciplined by the model's steady-state behavior and common choices for "deep" parameters.

The calibration used can be found in Table 2. The choices for "deep" parameter sare standard. The discount rate β is chosen to be 0.99 so that the annual risk-free return is 4%. Labor share α is set to two-thirds, and the labor disutility ζ is such that agents spend a third of non-sleeping time working in the steady-state. The Frish elasticity of labor supply υ_n is to set to the middle of a range of estimates in Chetty et al. (2011) , which studies both micro and macro evidence. For parameters related to inventory variables, I follow closely the strategy of modern inventory literature. Two parameters, the shape parameter of the idiosyncratic demand shock σ_v and the elasticity of substitution across varieties ρ, affect the markup and stockout probability at the variety level. Therefore, they are jointly calibrated to generate a 20% markup (a common choice see e.g. Galí (2015)) and a 5% stockout probability (according to findings in Bils (2004)) at the steady-state. The depreciation rate of inventory stock δ_e follows Wen (2011) to match the 6% annual depreciation. The stochastic process for the TFP shock is estimated from the publicly available series of US TFP from the Federal Reserve Bank of San Francesco (see Fernald (2014)).

Parameters concerning the product search friction deserve more attention. The elasticity of search effort in the matching function is quite elusive as households' search effort cannot be directly measured. It's commonly calibrated to satisfy the Hosios condition (Hosios (1990)) or to match indirect targets such as capacity utilization rate. (see e.g. Bai et al. (2012), den Haan (2013), and Petrosky-Nadeau and Wasmer (2015)). I take a more direct approach by utilizing one important estimate in Broda and Weinstein (2010). Using a dataset covering grocery stores in a large sector of US economy spanning from 1994 to 2003, Broda and Weinstein (2010) report that households consume 0.35% more unique products when their shopping time increase by 1%. Assuming shopping time is proportional to search effort in my model, I calibrate the elasticity of search effort ι such that when log-linearly approximated around steady state, the same relationship between search effort and varieties consumed is satisfied. Lastly, I calibrate the search disutility ξ to generate a steady-state value of d that is equivalent to 0.7 hours worth of hourly wage, which is taken from the Bureau of Labor Statistics' time-use survey as reported in Petrosky-Nadeau et al. (2016).

Table 2: Calibration

Parameter	Name	Value	Target	Source
β	Discount Rate	0.99	4% Return	
υ_n	Labor Elasticity	0.75	Frish Elas.	Chetty et al. (2011)
α	Labor Share	0.67		
ζ	Labor Disutility	1.5	1/3 time worked	ATUS
$\underline{v}$	Loc. v_i	0.04	Mean 1	
σ_v	Shape v_i	1.05	S.O. Prob=5%	Bils (2004)
ρ	Elas. of Subs.	1.17	20% markup	Galí (2015)
δ_e	Deprec. Inven,	0.015	6% annual	Wen (2011)
ι	Match Elasticity	1.18	0.35 elas.	Broda and Weinstein (2010)
ξ	Search Disutility	0.01	0.7 hr shopping[1]	Petrosky-Nadeau et al. (2016)
ρ_A	TFP Pers.	0.96		SF-FED TFP[2]
σ_A	TFP Vola.	0.02		

[1] Based on the American Time Use Survey, they document the average shopping time to 42 mintues per day over the 2003-2013 sample period.

[2] Estimated from the published US TFP series from the Federal Reserve Bank of San Francesco. See Fernald (2014) for more details.

Table 3: Dynamic Behavior of The Model

Statistics	Data[1]	Model[2]
Correlation($\frac{\text{Inven. Inves.}}{\text{GDP}}$,GDP)	0.66	0.58
AR(1) Coefficient of $\frac{\text{Inven.}}{\text{Sales}}$	0.75	0.89
Correlation($\frac{\text{Inven.}}{\text{Sales}}$,GDP)	-0.43	-0.30
Skewness($\frac{\text{Inven. Inves.}}{\text{GDP}}$)	-0.30	-0.46
Peak-to-trough Share of Inventory Investment	72%	54%
Trough-to-peak Share of Inventory Investment	8%	25%

NOTE: Data refers to US quarterly data taken from FRED database. Output is Real GDP and II is the Change In Private Inventory. See appendix for data sources. IS ratio is the inventory-sales ratio. See text for the model counterparts. AR(1) stands for estimated persistent coefficient when the series is fitted to a AR(1) process.

4.3 Performance

The first three rows of Table 3 are generally agreed upon[11] stylized facts about inventory dynamics: procyclical inventory investment, persistent inventory-sales ratio, and countercyclical inventory-sales ratio. The model is broadly consistent with these three stylized facts. Next, I will examine whether the model's dynamic behavior is consistent with the newly documented stylized facts.

4.3.1 Fact 1: Empirical Asymmetry of Inventory Investment

To examine the asymmetry of inventory dynamics in the model, I need to identify recessions and expansions in a way that is consistent with the data. In the US, recessions are dated by the National Bureau of Economic Research. Its methodology of dating recessions is not public knowledge. To mimic as close as possible what the NBER is doing, I treat the GDP growth rates simulated by the model as the demeaned growth rates of GDP in the data. In particular, I follow the following steps to identify recessions:

1. (US Data) Compute the average growth rates of GDP in recessionary quarters announced by the NBER.
2. (US Data) Compute the difference between average recessionary quarterly growth rates and the unconditional average.
3. (Model) Define recessions in the simulated data to be at least two consecutive periods whose GDP contraction rates are as large as the difference computed in the previous step

Following this method, the model's recessions last around four quarters on average and expansions 13 quarters. The US post-war recessions last 11.1 months on average and expansions 58.4 months[12]. In other words, both the model and the data spend around 20% of the time in recessions. Therefore this dating scheme performs reasonably well in dating the US business cycles. Inventory investment accounts for 54% of GDP declines (see the Table 3) in the model's recessions. In contrast, it only accounts for 25% of GDP increases in the model's expansions.

[11] Authors sometimes emphasize different set of stylized facts but these three seem to be the usual suspects. See e.g. Blinder (1986); Ramey and West (1999); Maccini et al. (2015)

[12] Taken directly from the NBER website here.

Comparing the skewness of inventory-investment-to-GDP ratio provides an alternative way to quantify the asymmetry of inventory investment dynamics. The results are presented in Table 3. Since inventory investment can turn negative, a common choice in the literature (e.g. Khan and Thomas (2007)) is to examine instead the ratio of inventory investment divided by output, as I do here in Table 3. The skewness of this ratio is stronger in the model (-0.46) than in the data (-0.30), but these two numbers are reasonably close. In sum, the model generates asymmetric shares of inventory investment in GDP movements.

4.3.2 Fact 2: Inventory-sales Ratio Lags GDP

In the model, inventory-sales lags GDP by five quarters which is similar to the four-quarter lag observed in the data (see Figure 4 and Figure 5). We can see from Figure 7 that the largest positive cross-correlation between inventory-sales ratio and GDP is at the fifth lag. To confirm the lagging relationship, I show in Figure 6 the contemporaneous inventory-sales ratio and the GDP five quarters ago from a simulated time series. I observe that these two time series exhibit a high correlation with a five-quarter lag while in the model they align well with a four-period lag (see Figure 4). Comparing Figure 5 and Figure 7, we can see that the cross-correlations between inventory-sales ratio and GDP exhibit similar profile across lags. This further supports the model's ability to match the inventory and GDP dynamics in the data.

In sum, the model is capable of broadly matching the two new stylized facts I documented. This is achieved under a reasonable parametrization of the model as described in Section 4.2. The next section seeks to understand how the addition of product market search enables a standard inventory model to generate these stylized facts.

5 Mechanism

To demonstrate the role played by product market search friction, I study the behavior of the model under different parameterizations of households' search disutility ξ. As we can see from Table 4, when ξ is lowered than the benchmark case of 0.01 (middle column), the search friction is weakened as evidenced by higher steady-state number of varieties in the first column. In this case, the asymmetry of inventory dynamics is weakened. First, inventory investment accounts for a smaller fraction of GDP contractions (second row) than the benchmark case. Meanwhile, inventory investment accounts for a higher fraction of

GDP expansions (third row) than the benchmark case. Second, the skewness of inventory-investment-to-GDP ratio is also less negative than the benchmark case. The opposite is true when I strengthen the product market search friction by increasing ξ (third column). In conclusion, a stronger product market search friction leads to a more pronounced inventory asymmetry.

Table 4: The Role of Product Market Search

		Benchmark	
Statistics	$\xi = 0.006$	$\xi = 0.010$	$\xi = 0.012$
Steady State Varieties, $\bar{x}$	0.91	0.88	0.75
Peak-to-trough Share	0.40	0.54	0.71
Trough-to-peak Share	0.29	0.25	0.12
Skewness($\frac{\text{Inven. Inves.}}{\text{GDP}}$)	-0.21	-0.46	-0.51

Note: The middle column represents the benchmark calibration. The first column shows the model's behavior when ξ is reduced to 0.006, which indicates a weakened product market search friction as search effort is less costly now. This is demonstrated by the higher steady state varieties consumed in the first row. The opposite is true for the third column where product market friction is strengthened. All numbers are averages of 10000 simulated time series with 200 quarters in length.

Since the variety good producers hold inventory, it is logical to analyze the main mechanism by understanding how product market friction affects their decision, especially their inventory decisions. To do so, I need to first define several useful intermediate variables. As a reminder, the variety i producer chooses the optimal price p_i and the optimal amount of goods available z_i. The cut-off point of demand shock realization above which stockout would happen (denoted by v_i^*) is then given by:

$$v_i^* = \frac{z_i}{\bar{c}} \left(\frac{p_i}{\bar{p}} \right)^{\frac{\rho}{\rho-1}} \tag{20}$$

where $\frac{z_i}{\bar{c}}$ represents the size of safety stock (goods available relative to the expected market size) and $\left(\frac{p_i}{\bar{p}} \right)^{\frac{\rho}{\rho-1}}$ captures the effect of price on demand. The return on holding inventory is defined by:

$$r^I \equiv \frac{(1-\delta_e)\mathbb{E}m' p_M'}{p_M}. \tag{21}$$

This variable captures how much $\frac{1}{p_M}$ units of inventory (cost exactly one numeraire) is valued by the producer when stockout doesn't happen: these inventories play no role in the current period but reduce $1-\delta_e$ units of new orders tomorrow. The producer values tomorrow's new order at its discounted market price $\mathbb{E}m'p_M'$. Lastly I define the markup to be $b_i \equiv \frac{p_i}{p_M}$.

Now we can intuitively understand optimality conditions Equation 11 (and Equation 12). Conditional on being matched with buyers, choosing a higher v_i^* means the producer is choosing a lower stockout probability $1 - F_v(v_i^*)$. Since $\rho > 1$, both increasing the availability (by making more new order y_i) and lowering price can decrease the probability of stockout. In turn, lowered stockout probability means the expected demand is more price elastic because when stockout happens the price elasticity is zero at the margin (increasing price infinitesimally would not change the quantity demanded because of the binding availability constraint). To demonstrate this intuitively, Figure 8 shows two demand curves when the realization of v_i is low (blue) and high (red). If the price is set to p^*, the variety producer is pricing at the zero elasticity region of demand when v_i is high enough, but at the positive elasticity region when v_i is low. Therefore, the expected price elasticity is a weighted average of zero and a positive constant (ρ), with higher weight given to ρ if the stockout is less likely and vice versa. In sum, the optimal markup (proportional to the inverse of price elasticity) is decreasing in v_i, which controls the stockout probability.

I can re-express Equation 11 to be:

$$b_i = \frac{\epsilon_i(v_i^*)}{\epsilon_i(v_i^*) - 1} r^I \tag{22}$$

and this negative relationship between markup and availability is depicted by the blue curve in Figure 9. Note that this relationship is convex. As z_i approaches zero, stockout is almost guaranteed to happen therefore the expected demand features close-to-zero price elasticity. As z_i approaches infinity, stockout is almost guaranteed to *not* happen thus the optimal markup approaches a finite and positive limit corresponding to no stockout.

The variety i producer faces another trade-off between the inventory holding cost and the opportunity cost of stockout. When the realization of v_i is too low to cause stockout, the goods available in excess of actual sales have no impact whatsoever on today's revenue. Its value is derived from its replacement cost tomorrow because unsold goods are carried over to next period as inventory stock. I can re-express Equation 14 as:

$$x\left[1 - F^v(v_i^*)\right]\left(b_i - r^I\right) = 1 - r^I. \tag{23}$$

which states that the optimal amount of goods available is such that the expected marginal profit when stockout happens equates the inventory holding cost $1 - r^I$.

The red solid curve in Figure 10 represents Equation 23. This curve is upward sloping for the following reason. Suppose b increases, then the missed out profit when stockout happens is larger. The producer would reduce the stockout probability $1 - F^v(v_i^*)$ by having more goods available. It follows that $z_i/\overline{c}$ must increase to restore equality. The intersection of these two curves jointly determines the markup b_i and the size of safety stock $z_i/\overline{c}$.

Now we are ready to explore how the variety i producer responds to changes in the probability to match with households x and the return on holding inventory r^I. Let's focus on the former first. Suppose x increases. If the producer doesn't change any of its decisions, then the LHS of Equation 23 is higher than the RHS because stockout is now more likely to happen. Therefore, for the same level of b_i, v_i^* has to be larger so that $1 - F^v(v_i^*)$ is smaller to restore equality. The result is the red curve shifting to the right in Figure 11. Intuitively, when the variety i producer finds it easier to be matched with households, the marginal benefit of having more goods available is higher as the expected stockout avoidance benefit is higher. The producer would then increase goods available until the stockout probability is lowered to the level justified by the return on holding inventory.

In an expansion, variety producers gradually lower their markups and increase their safety stocks. The opposite happens in a recession. We can see this phenomenon in the impulse responses to the productivity shock in Figure 14, as the productivity shock is the sole driver of fluctuations in the model. How is this related to the asymmetry of inventory dynamics? The key is that the producers' responses to shocks are dependent on their current choices of markups and safety stock sizes.

The effects of movements in matching probability x are asymmetric. This asymmetry arises from the curvature of Equation 22. Inspecting Figure 11, we can see that if x increase when the producer is choosing a high markup and a small amount of safety stock (point A in the figure), then the optimal response of the producer is to lower markup and enlarge its safety stock (intersection moves from A to A'). This scenario is consistent with what happens when a good productivity shock hits while the model economy is around a business cycle trough. Analogously, if x decreases when the producer is charging a lower markup and holding a large safety stock (point B), then the optimal response is to shrink its safety stock and increase markup (intersection moves from B to B'). However, the optimal markup curve (blue) is steeper in the high-markup-low-safe-stock region than the low-markup-high-safe-stock region. It follows that for two similarly-sized movements in x, the response at a business cycle peak (B to B') results in a larger response in safety stock than that at a business cycle trough (A to A'). This mechanism also explains why the markup level in the model is positively skewed as in the data. Producers' optimal markup is bounded from below and high markups frequently happen when producers shrink their safety stocks. All in all, variety producers' responses to movements in the matching probability is state dependent, and thus generates the asymmetric dynamics of stock and markup.

Movements in r^I could generate similar asymmetries in markup and safety stock movements, but their effects are dampened by an upward shift in the curve representing optimal markup (blue curve in Figure 13). Both increases in r^I and x lead to larger safety stock and lower markup. Since both the matching probability x and return on holding inventory r^I are procyclical (see Figure 15), the introduction of product market search (and hence endogenous x) enhances the difference of markup/stock trade between peaks and troughs, which in turn enhances the asymmetry of responses.

All in all, procyclical search effort from the households (buyers) endogenously increase the probability of stockout and thus affects the trade-off between adjusting inventory levels and adjusting prices. At the business cycle peak, the variety firm is holding a substantial amount of inventory in excess to expected sales as "safety stock" because it is easy to find customers, and in turn stockouts are prone to happen. If the economy starts to contract then households' reduced search effort makes it difficult for the variety producer to find customers. Firms lower the amount of safety stock relative to expected sales since the marginal benefit of holding inventory is significantly lower due to a large initial safety stock. At the same time, firms raise their markup as an alternative way to reach their target level of stockout probability. In relative terms, firms substitute higher markup for larger safety stock in managing stockout, which leads to large liquidation of inventory stocks following the shock. The opposite happens at the trough if an expansionary productivity shock hits the economy.

Product market search also enhances the model's ability to generate the lagging relationship between inventory-sales ratio and GDP. As I have established, both movements in the return on holding inventory r^I and matching probability x affect the variety producers' pricing and inventory decisions. However, these two forces peak at different times thus the expansion(contraction) of inventory stock is hump-shaped.

Figure 15 depicts the impulse responses of return on inventory holding, matching probability, and inventory stock after an expansionary productivity shock hits at the steady state. The return on holding inventory (dashed line) decreases initially but rises persistently above steady state afterwards. The peak is reached around 15 quarters from impact. In a standard model without endogenous matching probability, the movement of inventory would trace the movement of r^I. In my model, the matching probability jumps up on impact and continues to increase for several periods. These two forces combine to yield a pronounced hump-shaped response of inventory stock. The increased matching probability is associated with higher sales as a result of households' increased consumption quantities and varieties, while the movements in the return on holding inventory are largely due to changes in marginal cost (p_M) which is not directly related to sales. Therefore, the increase in sales is weakened as the product market search activity returns to normal while inventory stock continues to expand due to the intertemporal substitution of marginal costs. All in all, the inventory-sales ratio displays a lag with respect to GDP which highly correlates with sales.

6 Conclusion and Future Research

This paper shows that once product market search friction is incorporated into a standard inventory model, it can generate two newly documented stylized facts that are essential to understanding inventory's role in business cycle fluctuations. I use recent empirical findings and inventory data to discipline the quantitative behavior of the model by matching steady-state targets. Overall, my results suggest that product market search is a promising area for inventory research, with the potential to enrich many macroeconomic studies.

In a companion paper (Chen, 2017) of mine, I find that the government expenditure shock has larger effects on output during downturns in a Markov-Switching VAR (Sims and Zha (2006)). I also find that asymmetric responses of inventory accumulation across regimes could contribute to the asymmetric effect of monetary policy shock. This finding is consistent with the theoretical mechanism proposed in this paper. Therefore, the model can have important policy implications and warrants further research.

References

Aruoba, S. B., J. Fernandez-Villaverde, and J. F. Rubio-Ramirez (2006): "Comparing solution methods for dynamic equilibrium economies," *Journal of Economic dy-*

namics and Control, 30, 2477–2508.

Bai, Y., J.-V. Rios-Rull, and K. Storesletten (2012): "Demand shocks as productivity shocks," *Federal Reserve Board of Minneapolis*.

Bils, M. (2004): "Studying price markups from stockout behavior," Tech. rep., mimeo.

Bils, M. and J. A. Kahn (2000): "What Inventory Behavior Tells Us about Business Cycles," *American Economic Review*, 90, 458–481.

Blinder, A. (1981): "Retail Inventory Behavior and Business Fluctuations," *Brookings Papers on Economic Activity*, 12, 443–520.

Blinder, A. S. (1986): "Can the Production Smoothing Model of Inventory Behavior Be Saved?" *The Quarterly Journal of Economics*, 101, 431–53.

Blinder, A. S. and L. J. Maccini (1991): "The resurgence of inventory research: what have we learned?" *Journal of Economic Surveys*, 5, 291–328.

Broda, C. and D. E. Weinstein (2010): "Product creation and destruction: Evidence and price implications," *The American economic review*, 100, 691–723.

Caplin, A. S. (1985): "The variability of aggregate demand with (S, s) inventory policies," *Econometrica: Journal of the Econometric Society*, 1395–1409.

Chen, L. (2017): "What Does Inventory Tell Us About Business Cycle Regimes?" *Working Paper*.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins," *The American Economic Review*, 101, 471–475.

den Haan, W. (2013): "Inventories and the role of goods-market frictions for business cycles," .

den Haan, W. J., G. Ramey, and J. Watson (2000): "Job destruction and propagation of shocks," *The American Economic Review*, 90, 482.

Dixit, A. K. and J. E. Stiglitz (1977): "Monopolistic competition and optimum product diversity," *The American Economic Review*, 67, 297–308.

Feldstein, M. and A. Auerbach (1976): "Inventory behavior in durable-goods manufacturing: The target-adjustment model," *Brookings Papers on Economic Activity*, 1976, 351–408.

Fernald, J. G. (2014): "A quarterly, utilization-adjusted series on total factor productivity," .

Galí, J. (2015): *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton University Press.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, capacity utilization, and the real business cycle," *The American Economic Review*, 402–417.

Hodrick, R. J. and E. C. Prescott (1997): "Postwar US business cycles: an empirical investigation," *Journal of Money, credit, and Banking*, 1–16.

Hosios, A. J. (1990): "On the efficiency of matching and related models of search and unemployment," *The Review of Economic Studies*, 57, 279–298.

Huo, Z. and J.-V. Ríos-Rull (2013): "Paradox of thrift recessions," Tech. rep., National Bureau of Economic Research.

Jung, Y. and T. Yun (2005): "Monetary policy shocks, inventory dynamics, and price-setting behavior," *Inventory Dynamics, and Price-Setting Behavior*.

Kahn, J. A. (1987): "Inventories and the volatility of production," *The American Economic Review*, 667–679.

——— (1992): "Why is production more volatile than sales? Theory and evidence on the stockout-avoidance motive for inventory-holding," *The Quarterly Journal of Economics*, 481–510.

Khan, A. and J. K. Thomas (2007): "Inventories and the Business Cycle: An Equilibrium Analysis of (S, s) Policies," *American Economic Review*, 97, 1165–1188.

King, R. G. and S. T. Rebelo (1999): "Resuscitating real business cycles," *Handbook of macroeconomics*, 1, 927–1007.

Krusell, P. and A. A. Smith, Jr (1998): "Income and wealth heterogeneity in the macroeconomy," *Journal of Political Economy*, 106, 867–896.

Kryvtsov, O. and V. Midrigan (2012): "Inventories, Markups, and Real Rigidities in Menu Cost Models*," *The Review of Economic Studies*, rds028.

Li, N. (2012): "An Engel curve for variety," *University of Toronto*.

Maccini, L. J., B. Moore, and H. Schaller (2015): "Inventory behavior with permanent sales shocks," *Journal of Economic Dynamics and Control*, 53, 290–313.

McConnell, M. M. and G. Perez-Quiros (2000): "Output Fluctuations in the United States: What Has Changed Since the Early 1980's?" *The American Economic Review*, 90, 1464–1476.

McMahon, M. F. (2012): "Inventories in motion: a new approach to inventories over the business cycle," .

Petrosky-Nadeau, N. and E. Wasmer (2015): "Macroeconomic dynamics in a model of goods, labor, and credit market frictions," *Journal of Monetary Economics*, 72, 97–113.

Petrosky-Nadeau, N., E. Wasmer, and S. Zeng (2016): "Shopping time," *Economics Letters*, 143, 52–60.

Pissarides, C. A. (2000): *Equilibrium unemployment theory*, MIT press.

Ramey, V. A. and K. D. West (1999): "Inventories," *Handbook of macroeconomics*, 1, 863–923.

Sarte, P.-D., F. Schwartzman, and T. A. Lubik (2015): "What inventory behavior tells us about how business cycles have changed," *Journal of Monetary Economics*, 76, 264–283.

Sims, C. A. and T. Zha (2006): "Were there regime switches in US monetary policy?" *The American Economic Review*, 96, 54–81.

Wen, Y. (2005): "Understanding the inventory cycle," *Journal of Monetary Economics*, 52, 1533–1555.

——— (2011): "Input and output inventory dynamics," *American Economic Journal: Macroeconomics*, 3, 181–212.

Figure 6: IS Ratio is Lagging GDP by Five Quarters (Model)

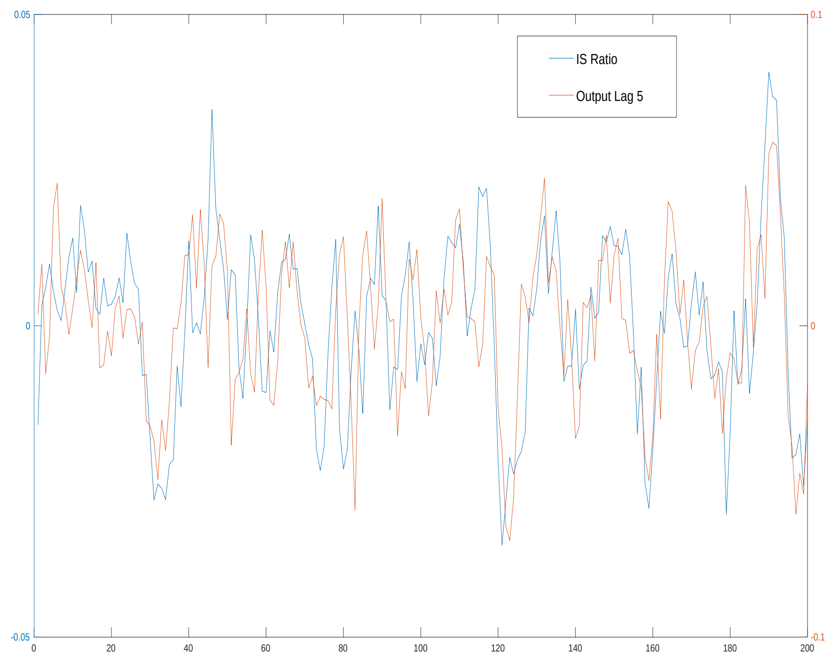


Figure 7: Cross-Correlation Between IS Ratio and GDP (Model)

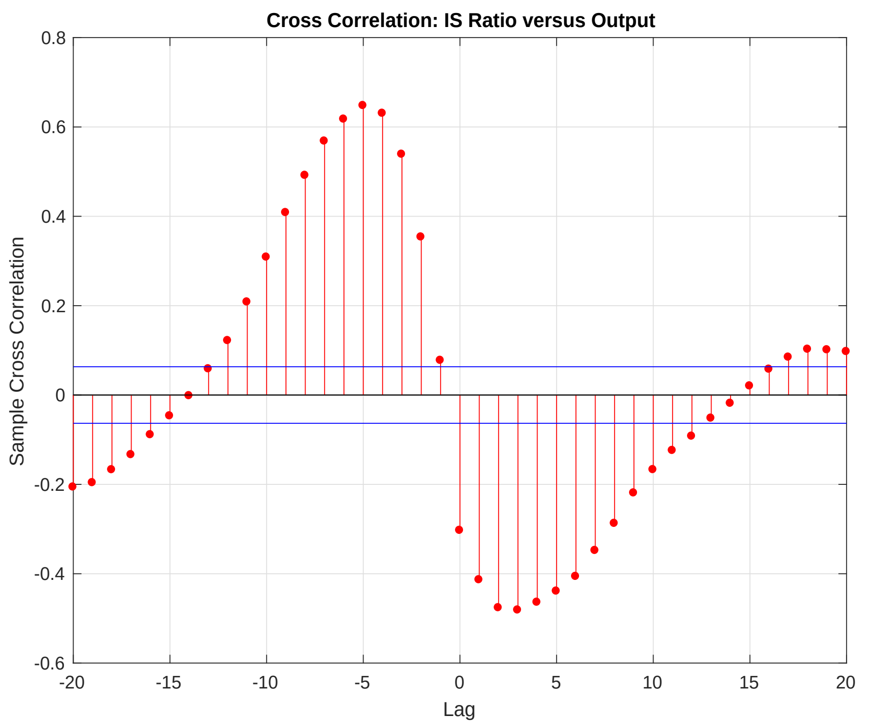


Figure 8: Demand Curve for Variety i

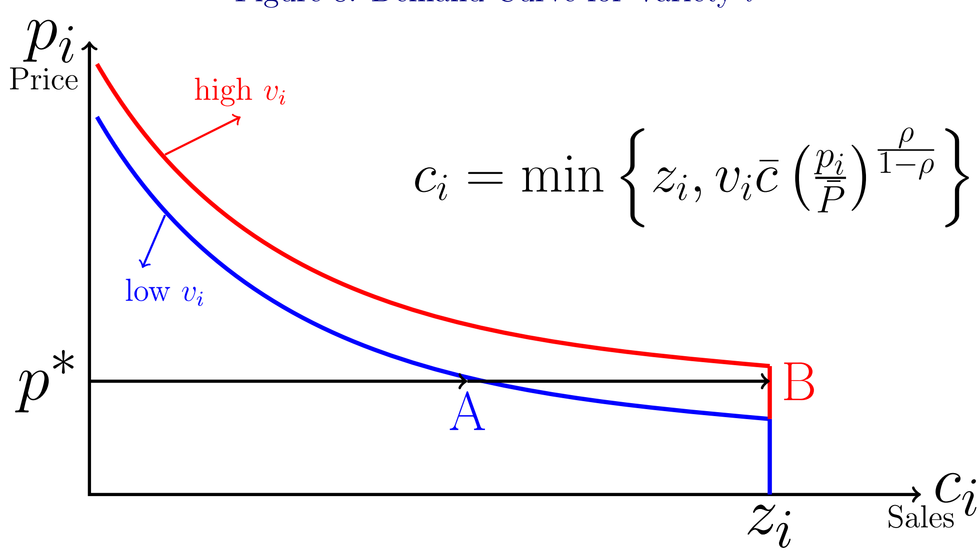


Figure 9: Optimal Choices For Markup

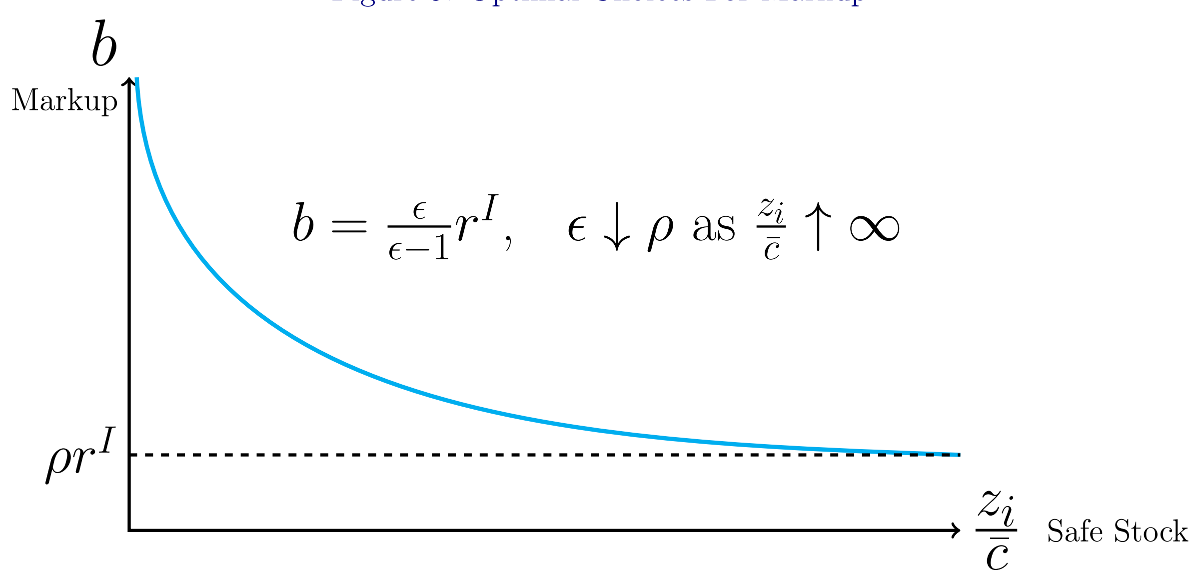


Figure 10: Joint Determination of Markup and Safe Buffer

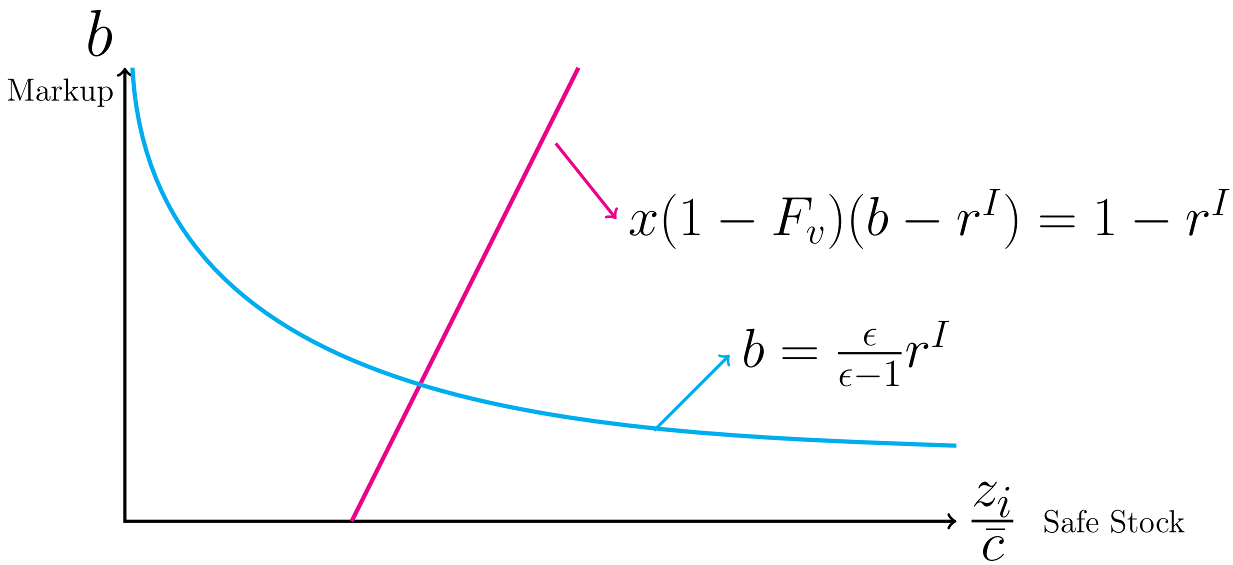


Figure 11: Shifting of x

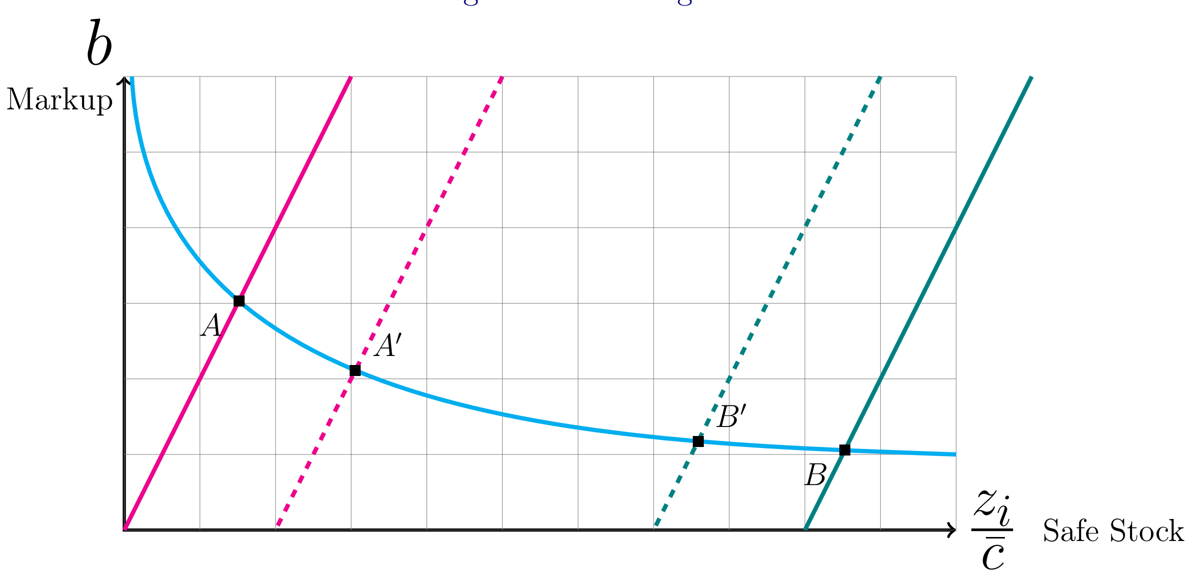


Figure 12: Joint Determination of Markup and Safe Buffer

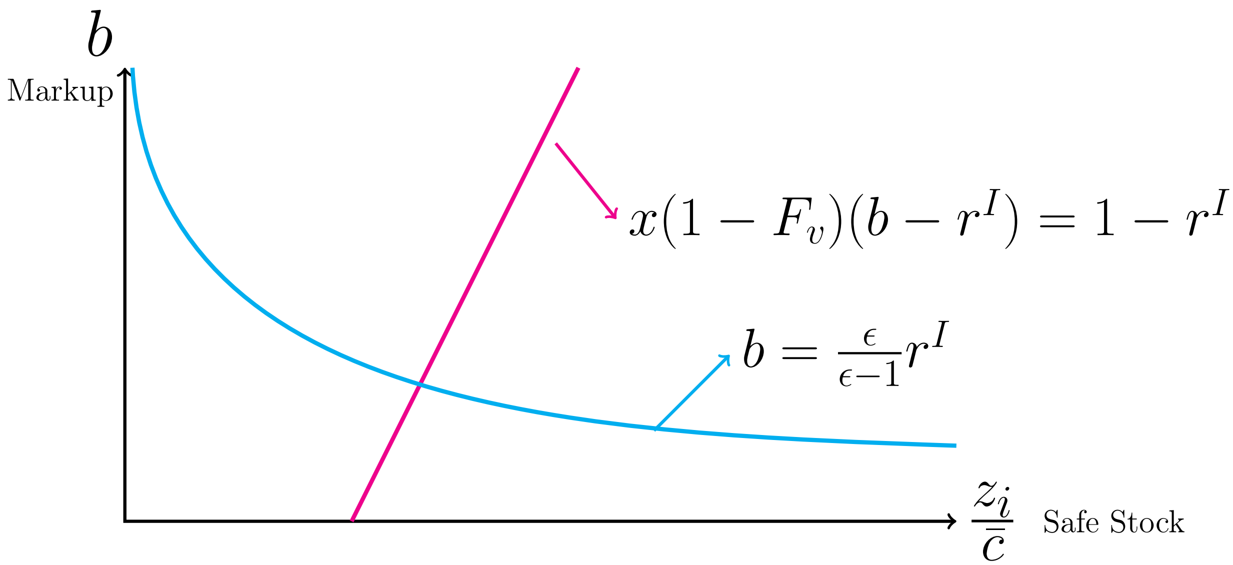


Figure 13: Increasing r^I

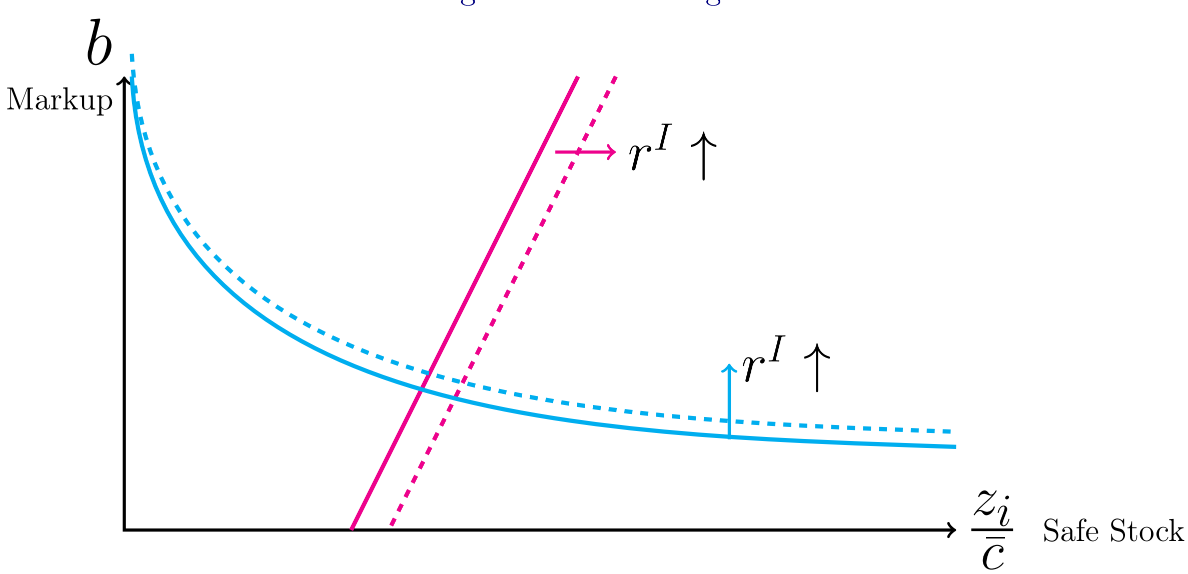
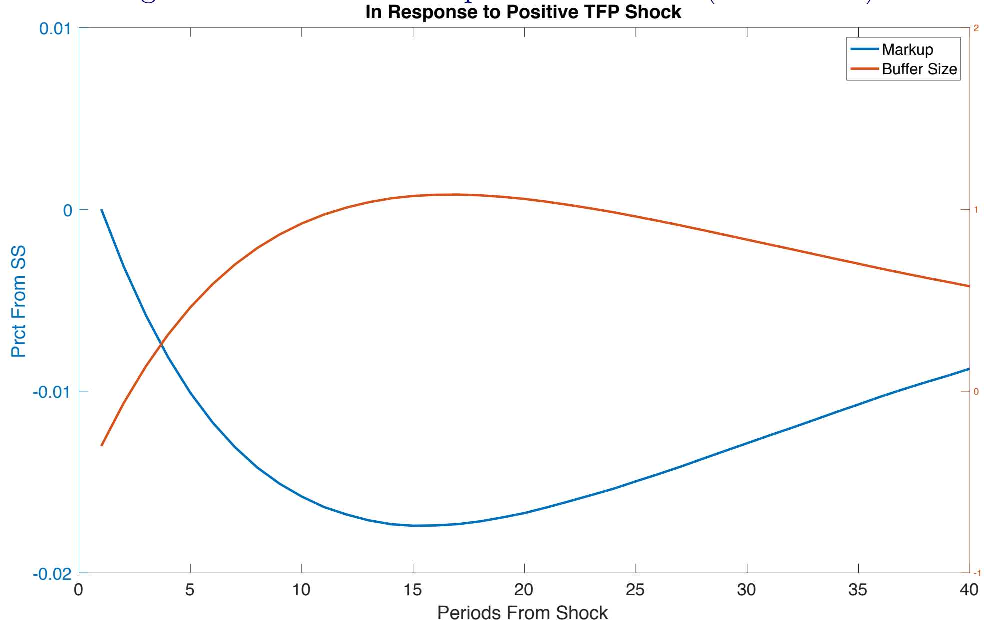
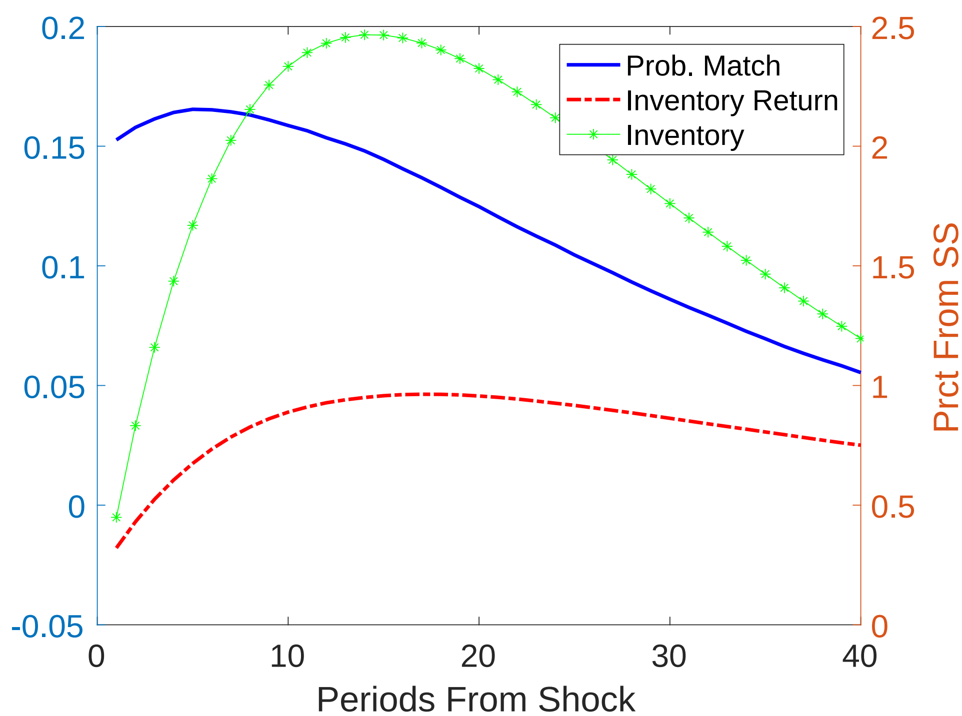


Figure 14: IRFs of Markup and Safe Stock (Buffer Size)



NOTE: Impulse Response Functions of Markups and Safe Stocks, Positive Productivity Shock

Figure 15: IRFs of Return on Inventory Holding, Matching Probability, and End-of-Period Inventory Stock



NOTE: IRFs of r^I, x, and e, Positive Productivity Shock

A Symmetric Equilibrium

A.1 Equilibrium Definition

The definition of a competitive equilibrium is given as the following:

DEFINITION. *Taking as given the exogenous process* $\left\{A_t, \{v_{i,t}\}_{i=0}^{1}\right\}_{t=0}^{\infty}$ *and initial inventory holding* e_{-1}*, a competitive equilibrium is a collection of stochastic processes*

$$\left\{c_t, \bar{c}_t, x_t, d_t, n_t, a_{t+1}, \Psi_{D,t}, p_t, w_t, p_t^*, p_{m,t}, z_t, \{c_{i,t}, y_{i,t}, p_{i,t}\}_{i=0}^{x_t}\right\}_{t=0}^{\infty}$$

such that

1. *Given prices, all agents solve their respective problems.*
2. *Product market results satisfy:*

$$x_t = G(d_t, 1) \tag{24}$$

3. *Intermediate goods market clears:*

$$F(A_t, n_t) = \int_0^{x_t} y_i di \tag{25}$$

4. *Variety goods market clears:*

$$c_{i,t} = \min\left\{z_{i,t}, v_{i,t}\left(\frac{p_{i,t}}{\bar{p}_t}\right)^{\frac{\rho}{1-\rho}}\bar{c}_t\right\}, \quad i \in [0, x_t] \tag{26}$$

A.2 First Order Equilibrium Conditions

The first order necessary conditions for the symmetric equilibrium is given as below:

Labor Supply:

$$\frac{w_t}{p_t} = \frac{-u_{n,t}}{u_{\tilde{c},t} x_t^{\rho-1}} \tag{27}$$

Consumption Euler Equation:

$$1 = \mathbb{E}m_{t+1}\left(1 + p_t c_t x_t - w_t n_t\right) \tag{28}$$

Search Effort:

$$\frac{-u_{d,t}}{u_{\tilde{c},t}} = (\rho - 1)c_t \frac{x_t^{\rho}}{d_t} \tag{29}$$

Stochastic Definition Definition:

$$m_{t+1} = \beta \frac{u_{\tilde{c},t+1} x_{t+1}^{\rho-1}/p_{t+1}}{u_{\tilde{c},t} x_t^{\rho-1}/p_t} \tag{30}$$

Matching:

$$x_t = \frac{d_t}{(1 + d_t^{\iota})^{1/\iota}} \tag{31}$$

Definition of Search Probability:

$$\Psi_{D,t} = \frac{x_t}{d_t} \tag{32}$$

Law of Motion for Inventory:

$$e_{t+1} = (1 - \delta_e)\left\{z_t\left[1 - x_t\left(1 - F_v(v_t^*)\right)\right] - x_t \mathscr{A}(v_t^*) c_t \left(\frac{p_t^*}{p_t}\right)^{\frac{\rho}{1-\rho}}\right\} \tag{33}$$

Cut-off Point Definition:

$$v_t^* = \frac{z_t}{c_t}\left(\frac{p_t^*}{p_t}\right)^{\frac{\rho}{\rho-1}} \tag{34}$$

Goods Availability Definition:

$$z_t = e_t + A_t n_t^{1-\alpha} \tag{35}$$

Optimal Price:

$$p_t^* = \rho \frac{1}{1 + [1 - F_v(v_t^*)] \frac{(\rho-1)v_t^*}{\mathscr{A}(v_t^*)}} (1 - \delta_e) \mathbb{E}_t m_{t+1} p_{M,t} \tag{36}$$

Optimal Goods Availability:

$$p_{M,t} = (1 - \delta_e) \mathbb{E}_t m_{t+1} p_{M,t} \left[1 - x_t(1 - F_v(v_t^*))\right] + x_t(1 - F_v(v_t^*)) p_t^* \tag{37}$$

Aggregate Price Index:

$$(p_t^*)^{\frac{1}{1-\rho}} \mathscr{A}(v_t^*) = p_t^{\frac{1}{1-\rho}} \left[1 - \mathscr{B}(v_t^*) \left(\frac{z_t}{c_t}\right)^{\frac{1}{\rho}}\right] \tag{38}$$

Input Price:

$$p_{M,t} = \frac{w_t}{A_t(1 - \alpha)} n_t^{\alpha} \tag{39}$$

where the following functional forms are assumed:

$$\mathscr{A}(v_t^*) \equiv \int_0^{v_t^*} v dF_v(v) = \frac{\sigma_v}{1 - \sigma_v} \left[\underline{v}^{\sigma_v} (v_t^*)^{1-\sigma_v} - \underline{v}\right] \tag{40}$$

$$\mathscr{B}(v_t^*) \equiv \int_{v_t^*}^{\infty} v^{1-\frac{1}{\rho}} dF_v(v) = \frac{\sigma_v \rho \underline{v}^{\sigma_v}}{1 - \rho + \rho\sigma_v} (v_t^*)^{\frac{\rho - 1 - \rho\sigma_v}{\rho}} \tag{41}$$

$$F_v(v_t^*) = 1 - \left(\frac{v_t^*}{\underline{v}}\right)^{-\sigma_v} \tag{42}$$

$$u_{\tilde{c},t} = \left(c_t x_t^{\rho} - \zeta \frac{n_t^{1+\upsilon_n}}{1 + \upsilon_n} - \xi d_t\right)^{-1} \tag{43}$$

$$u_{d,t} = -\xi \left(c_t x_t^{\rho} - \zeta \frac{n_t^{1+\upsilon_n}}{1 + \upsilon_n} - \xi d_t\right)^{-1} \tag{44}$$

$$u_{n,t} = -\zeta n_t^{\upsilon_n} \left(c_t x_t^{\rho} - \zeta \frac{n_t^{1+\upsilon_n}}{1 + \upsilon_n} - \xi d_t\right)^{-1} \tag{45}$$